Critical Parameter Selection in the Vibration Suppression of Large Space Structures

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This paper addresses the application of parameter sensitivity analysis to large flexible space structure models with uncertain parameters such as modal dampings, modal frequencies, and mode shape slopes at actuator (sensor) locations. A parameter ranking criterion is given to delineate the critical parameters in linear regulator problems. The quantitative measure is labeled "Parameter Error Index." Explicit and simple analytical formulas are obtained in terms of the modal data for the vibration suppression problem of large space structures. The proposed procedure is applied to the "Purdue Model," a generic two-dimensional large space structure model. Results indicate that, for the specific model used for the uncertainty of the parameters, modal dampings, modal frequencies, and mode shape slopes are critical in that order (in a broad sense). This type of information is useful in parameter estimation, robust control design, structure redesign, etc.

Nomenclature

I_α	$= \alpha \times \alpha$ identity matrix
k	= number of output variables
K	= stiffness matrix
l	= number of noise variables
m	= number of control variables
m_a	= number of actuators
M	= mass matrix
n	= number of states
N	= number of modes considered
p	= vector of uncertain parameters
$\bar{p}_i$	= nominal value of the uncertain parameter p_i
q	= vector of generalized coordinates q_i
r	= number of uncertain parameters
R^α	= real vector space of dimension α
t	= time variable
$\mathfrak{I}$	= transformation matrix
u	= control vector
u_c	= vector of control forces (torques) by actuators
u_w	= vector of actuator noise variables
V	= Quadratic Performance Index
w	= noise vector (external disturbance)
x	= state vector
y	= vector of controlled variables (output vector)
δ	= Dirac delta
ϵ	= belongs to
ζ_i	= modal damping of mode i
η	= vector of modal coordinates η_i
σ_i^2	= variance of parameter p_i
ω_i	= modal frequency of mode i
$[\dot{\ }]$	$= \dfrac{d[\ ]}{dt}$
$[\ddot{\ }]$	$= \dfrac{d^2[\ ]}{dt^2}$
$[\ ]^T$	= transpose of $[\ ]$
$[\tilde{\ }]$	= block diagonal $(\ldots[\ ]\ldots)$
$E[\ ]$	= expected value operator
$[\ ]_{pi}$	$= \dfrac{\partial}{\partial p_i}\Big\vert_{p_i=\bar{p}_i}$
$([\ ]_p^T)$	$= ([\ ]_{p1}^T [\ ]_{p2}^T \cdots [\ ]_{pr}^T)$

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I. Introduction

THE sensitivity of a dynamic system to variations of its parameters has been one of the basic themes in the treatment of dynamic systems. The uncertainty of the parameters of the model, compounded with the stringent requirements on performance, often make it mandatory to incorporate parameter sensitivity considerations into the control design process and in some applications to reduce the sensitivity to a desirable extent. In large-scale uncertain systems (systems with large dimensionality) the number of uncertain parameters can be quite high. The mathematical models of large flexible space structures form one example where the modal frequencies, mode shapes, and modal dampings for a large number of modes (hundreds of modes) are not precisely known. Control design methods which attempt to accommodate parameter sensitivity[1-3] tend to be quite complex. However, it is important to realize that all of the uncertain parameters in a dynamical system may not be equally critical or important to the performance task at hand; some may be more critical than others. Thus, to aid the control design process, it is helpful to first delineate the relative criticality of the uncertainty of the parameters to a given performance objective. In this paper this is accomplished by evaluating the performance degradation due to the uncertainty of each of the parameters. The quantitative measure associated with this is labeled the Parameter Error Index (PEI). Simple analytical expressions of PEI are obtained in terms of the modal data for the vibration suppression criterion of Large Space Structure (LSS) models. The proposed procedure is then applied to a generic two-dimensional LSS model called "Purdue Model"[4] and the results are discussed.

II. Parameter Ranking Criterion

Consider the linear time-invariant system

$$\dot{x}(t,\bar{p}) = A(\bar{p})x(t,\bar{p}) + B(\bar{p})u(t,\bar{p}) + D(\bar{p})w, \qquad x(0) = x_0 \tag{1a}$$

$$y(t,\bar{p}) = C(\bar{p})x(t,\bar{p}) \tag{1b}$$

where the state vector $x \epsilon R^n$, the control $u \epsilon R^m$, the process noise $w \epsilon R^l$, and the outputs y (the variables we wish to control) ϵR^k. The system matrices A, B, C, and D, are assumed to be continuous functions of constant but uncertain parameters $p_1, p_2, \ldots p_r$ with nominal values $p_i = \bar{p}_i$ $(i=1,2,\ldots,r)$. Let the vector of uncertain parameters be denoted by p, where

$$p^T = [p_1, p_2, \ldots, p_r] \tag{2}$$

The initial condition vector x_0 is a zero mean random vector with variance $E[x_0 x_0^T] = X_0$ and is assumed to be independent of the uncertain parameters. The disturbance w is assumed to be zero mean white noise process with intensity W. The vectors x_0 and w are assumed to be uncorrelated with each other. The uncertainty of the parameter vector p is characterized by

$$E(p-\bar{p}) = 0 \tag{3a}$$

$$E[(p-\bar{p})(p-\bar{p})^T] = \mathrm{diag}[\sigma_1^2 \; \sigma_2^2 \sigma_r^2], \qquad \sigma_i^2 > 0 \tag{3b}$$

i.e., σ_i^2 is the variance of the parameter p_i (when the parameter is certain $\sigma_{pi}^2 = 0$).

Let the performance index for the system be given by

$$V = \lim_{t\to\infty} \frac{1}{t} \int_0^t E[y^T(\tau) Q y(\tau) + u^T(\tau) R u(\tau)] \mathrm{d}\tau \tag{4}$$

where $Q \epsilon R^{k\times k}$ and $R \epsilon R^{m \times m}$ are symmetric, positive-definite matrices. In this expression, as discussed in Ref. 1, the expected value is taken over the probability distributions of y and u that are derived from both the distributions of the random process w and the random vector p. Since w and p are independent, the expected value of a function of y and u is

$$E[g(y,u)] = \int_w E_{(p)} p_r(w) \mathrm{d}w = \int_p E_{(w)} p_r(p) \mathrm{d}p \tag{5}$$

where

$$E_{(p)} = \int_p g(w,p) p_r(p) \mathrm{d}p \tag{6}$$

is the expected value over the distribution of p and

$$E_{(w)} = \int_w g(w,p) p_r(w) \mathrm{d}w \tag{7}$$

is the expected value over the distribution of w and $p_r(\cdot)$ denotes the probability distribution of $(\cdot)$. We refer to system (1) along with Eq. (4) as the "nominal system." Henceforth, the explicit time and parameter dependence of x, u, y, V, and other related quantities will not be shown (except when deemed necessary).

The system given by Eq. (1) can be rewritten, either for the open-loop case (i.e., $u \equiv 0$, when the dimension of the system is too large to determine u) or in the closed-loop case with a feedback control law

$$u = Gx \tag{8}$$

as

$$\dot{\mathfrak{x}}(\bar{p}) = \mathfrak{a}(\bar{p}) \mathfrak{x}(\bar{p}) + D(\bar{p}) w \tag{9a}$$

$$\mathfrak{y}(\bar{p}) = \mathfrak{C}(\bar{p}) \mathfrak{x}(\bar{p}) \tag{9b}$$

and the performance index Eq. (4) becomes

$$V(\bar{p}) = \lim_{t\to\infty} \frac{1}{t} \int_0^t E[\mathfrak{y}^T(\bar{p}) \mathfrak{Q} \mathfrak{y}(\bar{p})] \mathrm{d}\tau \tag{10}$$

where $\mathfrak{y}^T = [y^T \; u^T]$, $\mathfrak{Q} = \mathrm{diag}[Q,R]$. [Note that for $u \equiv 0$, the symbols in relations (1) and (4) become identical to the ones in relations (9) and (10).] We assume the matrix $\mathfrak{a}$ to be stable. Also the matrix pair $[\mathfrak{a}, D]$ is assumed to be controllable and the pair $[\mathfrak{a}, \mathfrak{C}]$ is assumed to be completely observable.

Let $\mathfrak{x}_{pi}$ and $\mathfrak{y}_{pi}$ denote the state sensitivity and output sensitivity, respectively, with respect to p_i. The state sensitivity vector x_p is denoted by the nr vector

$$(\mathfrak{x}_p)^T = [\mathfrak{x}_{p1}^T, \mathfrak{x}_{p2}^T, \ldots, \mathfrak{x}_{pr}^T] \tag{11}$$

The output sensitivity vector $\mathfrak{y}_p \epsilon R^{kr}$ is similarly defined.

With notation given in the Nomenclature, the sensitivity subsystem for nominal system (1) can be described by[3]

$$\dot{\mathfrak{x}}_p = \mathfrak{a}_p \mathfrak{x} + \tilde{\mathfrak{a}} \mathfrak{x}_p + D_p w, \qquad \mathfrak{x}_p(0) = 0 \tag{12a}$$

$$\mathfrak{y}_p = \mathfrak{C}_p \mathfrak{x} + \tilde{\mathfrak{C}} \mathfrak{x}_p \tag{12b}$$

Parameter Error Index

When parameter uncertainty is taken into account, one can expand $\mathfrak{y}(p)$, the output at the actual values of the uncertain parameters, about the nominal value $\mathfrak{y}(\bar{p})$ as

$$\mathfrak{y}(p) = \mathfrak{y}(\bar{p}) + \sum_{i=1}^{r} (p_i - \bar{p}_i) \mathfrak{y}_{pi} + \text{higher order terms} \tag{13}$$

Assuming small parameter variations, we can write

$$\mathfrak{y}(p) \cong \mathfrak{y}(\bar{p}) + \sum_{i=1}^{r} (p_i - \bar{p}_i) \mathfrak{y}_{pi} \tag{14}$$

Correspondingly, the performance index V can be written, again to first-order accuracy, as

$$\begin{aligned} V(p) &\cong \lim_{t\to\infty} \frac{1}{t} E\left[\int_0^t \mathfrak{y}^T(\bar{p}) \mathfrak{Q} \mathfrak{y}(\bar{p}) + \sum_{i=1}^{r} \mathfrak{y}_{pi}^T \sigma_i^2 \mathfrak{Q} \mathfrak{y}_{pi}\right] \mathrm{d}\tau \\ &\cong V(\bar{p}) + \Delta V(\bar{p}) \end{aligned} \tag{15}$$

where the expectation operator is now over the random process w with $\mathfrak{y}(\bar{p})$ and $\mathfrak{y}_{pi}$ satisfying the model given by relations (9) and (12). Note that a sufficient condition for the first-order analysis to be valid is that the parameter variations (and thus the variances of the parameters σ_i^2) be small. A typical assumption made is that the variations are small if they are about one order of magnitude smaller than the nominal values.

Noting that $\Delta V(\bar{p})$ is >0, we label $\Delta V(\bar{p})$ as the performance degradation caused due to the uncertainty of the parameters. The contribution of each of the parameters is given by

$$\Delta V_i \triangleq V(\bar{p}_i) = \lim_{t\to\infty} \frac{1}{t} E \int_0^t \mathfrak{y}_{pi}^T \sigma_i^2 \mathfrak{Q} \mathfrak{y}_{pi} \tag{16a}$$

where

$$\Delta V(\bar{p}) = \sum_{i=1}^{r} \Delta V_i \tag{16b}$$

The quantity ΔV_i is labeled as PEI. A descending-order sequence of the PEI establishes the relative "criticality" of the parameters to the performance objective, i.e., the significant parameters are determined by their ranking in the manner

$$\Delta V_1 \geq \Delta V_2 \geq V_3 \geq \ldots \geq \Delta V_r \tag{16c}$$

From relations (9), (12), and (16), we observe that the PEI are obtained by the relationship[5,6]

$$\Delta V_i = \text{trace}\,[P_{i11}\mathcal{C}_{pi}^T\sigma_i^2\mathcal{Q}\mathcal{C}_{pi} + P_{i12}\mathcal{C}^T\sigma_i^2\mathcal{Q}\mathcal{C}_{pi} + P_{i12}\mathcal{C}_{pi}^T\sigma_i^2\mathcal{Q}\mathcal{C} + P_{i22}\mathcal{C}^T(\sigma_i^2\mathcal{Q})\mathcal{C}] \tag{17}$$

where

$$P_{i11}\mathfrak{a}^T + \mathfrak{a}P_{i11} + DWD^T = 0$$

$$P_{i11}\mathfrak{a}_{pi}^T + P_{i12}\mathfrak{a}^T + \mathfrak{a}P_{i12} + DWD_{pi}^T = 0$$

$$P_{i12}^T\mathfrak{a}_{pi}^T + \mathfrak{a}_{pi}P_{i12} + P_{i22}\mathfrak{a}^T + \mathfrak{a}P_{i22} + D_{pi}WD_{pi}^T = 0$$

Thus in the normal case the computation of PEI's involves the solution of "3r" nth order Liapunov equations. However the extension of the above analysis to the open-loop LSS models has some significant simplifications. The attractive feature associated with this case is that we can get explicit formulas for the PEI in terms of the modal data so that we can easily compute PEI's for a larger number of parameters. The details are discussed in Sec. III. A different case of simplification (other than LSS models) is given in Ref. 6.

III. Critical Parameter Selection for LSS Models

In this section, attention is focused on the following matrix second-order system which represents a broad class of flexible spacecraft control problems, in the linear range

$$M\ddot{q} + Kq = B'u \tag{18}$$

Here M, K are $N \times N$; u is $m_a \times 1$ and the dimensions of the remaining matrices are readily inferred. We further assume that

$$M^T = M > 0, \qquad K^T = K > 0 \tag{19}$$

where $K>0$ indicates that we have set aside the rigid body modes of the structure and have retained only the elastic modes for parameter reduction. A transformation $\mathfrak{I}$ exists that simultaneously diagonalizes M and K

$$\mathfrak{I}^T M = I_N, \qquad \mathfrak{I}^T KH = \omega^2 \tag{20}$$

where ω^2 is diagonal. Then with

$$q = \mathfrak{I}\eta \tag{21}$$

the system of Eqs. (18) and (19) transforms to

$$\ddot{\eta} + \omega^2\eta = Bu \tag{22a}$$

where

$$B = \mathfrak{I}^T B' \tag{22b}$$

The modal frequencies of vibration of system (18) are ω_i with

$$\mathfrak{I} = \iota_1, \iota_2, \ldots, \iota_N \tag{23}$$

Let $u = u_c + u_w$ where u_c represents the desired control input and u_w represents the noisy distrubance emanating from the actuators (electronic noise, vibrations, etc.). u_w is modeled as a zero mean white noise process with intensity $W \triangleq E[u_w(t)u_w^T(t)]$. It is presumed that the order of Eq. (18) is too high for the control u_c to be known a priori. Thus we take $u_c(t) \equiv 0$ in this application.

If we are dealing with a physical structure, then some damping exists in the structure. We arbitrarily add modal damping to Eqs. (22) so that the system of interest is

$$\ddot{\eta} + 2\zeta\omega\dot{\eta} + \omega^2\eta = Bu_w$$

$$2\zeta\omega = \text{diag}(2\zeta_1\omega_1, \ldots, 2\zeta_N\omega_N) \quad \omega^2 = \text{diag}[\omega_1^2, \omega_2^2, \ldots, \omega_N^2] \tag{24}$$

For the vibration suppression problem, the performance metric taken is the sum of potential and kinetic energies, i.e.,

$$V = \lim_{t\to\infty} E \sum_{i=1}^{N} [\eta_i\omega_i^2 + \dot{\eta}_i^2] \tag{25}$$

In the above model, the modal dampings ζ_i are perhaps the least accurately known parameters. Modal damping of the order of $\zeta_i = 0.005$ are typically assumed for space structures. It is to be recognized that the mass and stiffness matrices are never known exactly. Uncertainties in the mass and stiffness matrices M and K lead to incorrect eigenvectors ι_i. This renders uncertain the mode shapes and mode slopes at actuator locations (in B) on the elastic portions of the structure. The structure of the matrix ω^2 also is altered by errors in $\mathfrak{I}$. It is plausible, then, that a reasonable set of uncertain parameters are

$$\{\omega_i, b_i^T \triangleq \iota_i^T B, \text{ and } \zeta_i\} \tag{26}$$

where b_i^T denotes the ith row of B.

Critical Parameters for the Vibration Suppression Problem

Given the system of Eq. (24), performance index equation (25), and the uncertain parameters of Eq. (26), we can apply the sensitivity analysis of Sec. II to the above model and obtain the PEI. The results are summarized in the following theorem:

Theorem 1:
Given the system

$$\ddot{\eta}_i + 2\zeta_i\omega_i\eta_i + \omega_i^2\eta_i = b_i^T u_w \qquad i = 1, \ldots, N \tag{27a}$$

$$E[u_w(t)] = 0, \qquad E[u_w(t)u_w^T(\tau)] = W\delta(t-\tau) \tag{27b}$$

and the performance criterion for vibration suppression

$$V = \lim_{t\to\infty} E\left[\sum_{i=1}^{N} (\omega_i^2\eta_i^2 + \dot{\eta}_i^2)\right] \tag{28}$$

then the PEI for the uncertain parameters ω_i, ζ_i and b_{ij}^T (the ijth element of b_i^T) are given by ($i = 1, \ldots, N$)

$$\Delta V(\omega_i) = \sigma_{\omega_i}^2 (2\zeta_i^2 + 1)(8\zeta_i^3\omega_i^3)^{-1} b_i^T W b_i$$

$$\Delta V(\zeta_i) = \sigma_{\zeta_i}^2 (4\zeta_i^3\omega_i)^{-1} b_i^T W b_i$$

$$\Delta V(b_{ij}^T) = \sigma_{b_{ij}}^2 (2\zeta_i\omega_i)^{-1} W_{jj}, \quad \Delta V(b_i^T) = \sum_{j=1}^{m_a} \Delta V(b_{ij}^T) \tag{29}$$

The above formulas are obtained by the direct solution of Eq. (17) with appropriate matrices specialized to Eqs. (27) and (28). In the present case drastic simplifications arise because of the simultaneous decoupling in the modal equations, performance index, and uncertain parameter sets, as evidenced by relations (27), (28), and (26), respectively. In other words, for each mode i, we have an independent set of equations to solve for. Thus the entire computation essentially reduces to solving simple 2×2 Liapunov equations in a generic fashion. For complete details of the computation one should consult Ref. 7.

In Refs. 8 and 9, the author (in association with Skelton) defined parameter costs in the context of component cost analysis[10] to determine the critical parameters. The PEI's defined here are slightly different from the parameter costs of Refs. 8 and 9. In Refs. 8 and 9, the performance index considered is

$$V_s = \lim_{t\to\infty} \frac{1}{t} E \int_0^t \left[y^T Q y + \sum_{i=1}^{r} y_{pi}^T \rho_i Q y_{pi} \right] \mathrm{d}\tau \tag{30}$$

where one choice for ρ_i could be $\rho_i = \sigma_i^2$, in which case V_s is the same as $V(p)$ of Eq. (15). The parameter cost is defined as

$$V(p_i) \triangleq \frac{1}{2} \lim_{t\to\infty} E\left[\frac{\partial V_s}{\partial x_{pi}} x_{pi} \right] \tag{31}$$

In other words, x_{pi} is the component of the higher-order sensitivity model. Because of the nature of definition, it is viewed that the parameter costs are suitable for model truncation or reduction (including sensitivity models), whereas PEI are suitable for parameter selection or truncation. The reason for this is that the entire information about parameter uncertainty (A_{pi}, D_{pi}, and C_{pi}) is reflected in the definition of PEI, wheras the information contained in C_{pi} is not reflected in the parameter costs. However, for the special case of $C=$ identity so that $C_{pi}=0$, the definition of both PEI and parameter cost turns out to be the same (as is the case with the application considered in Refs. 8 and 9). For the sake of completeness we reproduce the expressions given for the shape (and attitude) control problem in Refs. 8 and 9 employing the definition of PEI. (Note: In this case ω_i^2 is taken as the uncertain parameter.)

PEI for the Shape and Attitude Control Problem

Given the performance index

$$V = \lim_{t\to\infty} E \sum_{i=1}^{N} [\eta_i^2 + \beta \dot{\eta}_i^2] \tag{32}$$

the PEI are given by

$$\Delta V(\zeta_i) = \sigma_{\zeta_i}^2 (8\zeta_i^3 \omega_i^3)^{-1} (1+\beta\omega_i^2) b_i^T W b_i \tag{33a}$$

$$\Delta V(\omega_i^2) = \sigma_{\omega_\theta}^2 \cup (\forall \cup \zeta_i^3 \omega_i^7)^{-1} (1+4\zeta_i^2+\beta\omega_i^2) b_i^T W b_i \tag{33b}$$

$$\Delta V(b_{ij}^T) = \sigma_{b_{ij}}^2 (4\zeta_i \omega_i^3)^{-1} (1+\beta\omega_i^2) W_{jj} \tag{33c}$$

Some discussion of the usefulness of the theorems is in order. First, if there is no parameter uncertainty in the problem, then it is appropriate to ignore parameter sensitivity and take $\sigma_i^2 = 0$ in Eq. (15). The cost function in Eq. (15) with $\sigma_i^2 > 0$ registers the analyst's concern over parameter uncertainty.

The parameter error indices are observed to be products of four properties of the modal data,

$\Delta V(p_i)$ = (parameter variance) (mode time cosntant)
= (mode observability) mode disturbability)

Thus a parameter might be critical even if it has a small variance σ_i^2, if disturbability and observability is high. Conversely, a large variance is not sufficient to guarantee that the parameter is critical since it might be associated with a mode of low observability or disturbability.

The cirtical parameters of a structure may now be determined by the simple calculation of Eq. (29) or (33)

Table 1 Parameter sequence (order of decreasing importance)

ζ_{10}	ζ_9,	ζ_7,	ζ_6,	ζ_5,	ζ_4,	ζ_3,	ζ_2,	ζ_1
ω_1,	ω_2,	ω_3,	ω_4,	ω_9,	ω_{10},	ω_5	ω_7,	ω_6
B_2,	B_4,	B_{10},	B_9,	B_7	B_3,	B_6,	B_1,	B_5

$$\Delta V(p_i) \geq \Delta V(p_2) \geq \ldots \Delta V(p_r) \tag{34}$$

Such information can be valuable in parameter identification problems (e.g., Which parameters should we identify?), and for making certain structure redesign suggestions (e.g., Which modes, if any, should we add passive damping in? For which modes, if any, should we attempt to increase the frequency?). Even if the structure is not redesigned, the PEI can be useful in modeling. For example, if the mode shapes at actuator j are critical parameters [by Eq. (34)] then it might be prudent to redistribute the elements of a finite element model of the structure so that the finite element grid is finer in the area of that actuator, yielding more accurate mode shape information at that location.

IV. Application to Purdue Model

We now apply the method described in the previous sections to the generic large space structure described in Ref. 4 to determine the critical parameters.

The Model

The nondimensional numerical model given below, which is a low order model of the "Purdue Model," consists of nine elastic modes and corresponds to the equation

$$\dot{x} = Ax + B_1 u_w, \qquad x \in R^{18}, \qquad u_w \in R^{10}$$

(N = number of modes considered = 9 and m_a = number of actuators = 10), where

$$x^T = [\eta^T \dot{\eta}^T]$$

$$\eta^T = [\eta_{e1} \eta_{e2} \eta_{e3} \eta_{e4} \eta_{e5} \eta_{e9} \eta_{e6} \eta_{e7} \eta_{e10}]$$

(Note: The modes[4] were ordered by modal cost analysis[10] and the absence of η_{e8} indicates that it did not survive the cost ordered truncation.)

$$A = \begin{bmatrix} 0 & I_{9\times 9} \\ -\omega^2 & -2\zeta\omega \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0_{9\times 10} \\ B' \end{bmatrix}$$

where $0_{9\times 10}$ is a (9×10) zero matrix.

$$\omega^2 = \mathrm{diag}\,[\omega_1^2 \omega_2^2 \omega_3^2 \omega_4^2 \omega_5^2 \omega_9^2 \omega_6^2 \omega_7^2 \omega_{10}^2]$$

$= \mathrm{diag}\,[4.1032E+02, 1.0897E+03, 3.2656E+03, 4.9503E+03, 1.0947E+04, 2.6194E+04, 1.4245E+04, 1.7060E+04, 3.1443E+04]$

$$2\zeta\omega = \mathrm{diag}\,[2\zeta_1\omega_1, 2\zeta_2\omega_2, \ldots, 2\zeta_{10}\omega_{10}]$$

$\zeta_i = 0.005$ for all i

The matrix B' is

$B' =$

−1.0573E−01	2.2062E+00	1.0573E−01	2.2062E+00	0	0	−1.0573E−01	−2.2062E+00	1.0573E−01	−2.2062E+00
3.0691E+00	−5.4806E−01	3.0691E+00	5.4806E−01	0	0	−3.0691E+00	5.4806E−01	−3.0691E+00	−5.4806E−01
4.4194E−01	3.7745E+00	−4.4194E−01	3.7745E+00	0	−2.8571E+00	4.4194E−01	3.7745E+00	−4.4194E−01	3.7745E+00
2.9777E+00	−2.0442E+00	2.9777E+00	2.0442E+00	−3.0626E+00	0	−2.9777E+00	2.0442E+00	2.9777E+00	2.0442E+00
2.1157E+00	4.7510E+00	−2.1157E+00	4.7510E+00	0	0	−2.1157E+00	−4.7510E+00	2.1157E+00	−4.7510E+00
−8.2330E+00	1.7454E+00	8.2330E+00	1.7454E+00	0	3.1422E+00	−8.2330E+00	1.7454E+00	8.2330E+00	1.7454E+00
2.6296E+00	4.4189E+00	2.6296E+00	4.4189E+00	0	0	−2.6296E+00	4.4189E+00	2.6296E+00	−4.4189E+00
3.4633E+00	3.1066E+00	−3.4633E+00	3.1066E+00	0	6.5744E+00	3.4633E+00	3.1066E+00	−3.4633E+00	3.1066E+00
8.6962E+00	8.3946E−01	−8.6962E+00	8.3946E−01	0	0	−8.6962E+00	−8.3946E−01	−8.6962E+00	−8.3946E−01

$$W = E[u_w(t)u_w^T(t)] \text{ is a } 10\times 10 \text{ matrix}$$

$$= \text{diag } [2.0\,\text{E}-14]$$

Characterization of Parameter Uncertainty

The modal dampings ζ_i, modal frequencies ω_i, the mode shapes of actuator locations, and $(B_i)_j$ of the above-listed modes are taken as the uncertain parameters. The uncertainty in these parameters tends to increase with mode number i. It is plausible, then, that a reasonable model for uncertainties in each of these parameters is a variance proportional to mode number $\sigma_i^2 = i\sigma_0^2$, where σ_i^2 is the variance associated with parameters in mode i, for some appropriate constant σ_0. Of course, more appropriate statistical models might be developed for particular structures. The above model is offered only to help put the relative degree of uncertainty of the parameters of the dynamic model into perspective.

For the present application, i.e., vibration suppression as the performance objective, we use

$$\sigma_0^2 = 1 \qquad \text{for } \omega_i^2 \text{ and } B_{ij}$$

$$\sigma_0^2 = 25\times 10^{-4} \qquad \text{for the damping } \zeta_i$$

Results and Discussion

Computing the PEI determined by the relationships given in Eq. (29) we find the critical parameters listed in Table 1. Some conclusions from Table 1 are as follows.

1) In a broad sense the critical parameters of the open-loop structure for the vibration suppression problem are in this order: modal dampings, modal frequencies, and mode slopes at actuator locations.

2) The lack of any definite pattern in the ranking of mode shape slopes indicates that the ranking is sensitive to the modeling of actuator noise characteristics (W_{jj} and $b_i^T W b_i$ terms).

3) The parameter ranking for the shape control problem[8,9] indicated (in a vey broad sense) that mode slopes are more critical than modal frequencies. (Note: In arriving at the conclusions in Refs. 8 and 9, the uncertainty in ω_i^2, not in ω_i, was considered.) However, the basic conclusion one can draw is that the performance objective does indeed influence the parameter ranking.

V. Conclusions

The paper presents a criterion by which one can delineate the "critical" parameters for a given performance objective in Linear Quadratic Gaussian regulator problems with uncertain parameters. The quantitative measure is labeled the "Parameter Error Index." When applied to large space structure models, this index can be expressed in simple explicit formulas in terms of the modal data. Application of this procedure to the vibration suppression problem of the "Purdue Model," a generic two-dimensional large space structure model, indicates that modal dampings, modal frequencies, and mode shape slopes are critical in that order.

In conclusion, it should be noted that the validity of these results is basically proportional to the validity of the uncertainty model one presumes for the uncertain parameters. When the uncertainties are large the results may not indicate the true picture. Of course, the method given can alsways be used when the uncertainties are small. In the example given, the first few modes are considered for analysis whose uncertainty can be assumed to be reasonably small. Thus, the main contribution of the paper is the presentation of a methodology with which one can delineate the critical parameters in a linear regulator problem. Further research is underway to extend the proposed concepts for more general situations and for different uncertainty models for the parameters.

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